

Doublet-Triplet Splitting in an $SU(5)$ Grand Unification

Toshifumi Yamashita*

*Maskawa Institute for Science and Culture, Kyoto Sangyo University,
Kyoto 603-8555, Japan*

Abstract

We present a new solution to the doublet-triplet splitting problem which also works in a supersymmetric $SU(5)$ grand unified theory and is testable at TeV-scale collider experiments. In our model, the $SU(5)$ symmetry is broken through the Hosotani mechanism. Thanks to the phase nature of the Hosotani-breaking, the “missing VEV” can be realized even in an $SU(5)$ model. A general and distinctive prediction of this solution is the existence of light adjoint chiral supermultiplets with masses of the supersymmetry breaking scale. Though these fields disturb the gauge coupling unification, it can be recovered keeping the unified gauge coupling constant perturbative by introducing additional vector-like particles, which may be also observed in the upcoming collider experiments.

*e-mail: tyamashi@cc.kyoto-su.ac.jp

1 Introduction

The grand unified theories (GUTs) [1] unify the three forces in the standard model (SM), leading to the beautiful unification among the quarks and leptons. In the supersymmetric (SUSY) version [2], where the so-called hierarchy problem is solved, the minimal model predicts the observed gauge coupling unification (GCU) in the minimal SUSY SM (MSSM), up to the threshold corrections. Thus, the SUSY-GUTs are regarded as interesting candidates for the physics beyond the SM.

They have, however, some awkward issues to address. The most serious one is the doublet-triplet (DT) splitting problem[†]. This is caused by the failure of unifying the SM Higgs field which forces to introduce its partner. In the minimal model, an $SU(5)$ model, its partner is color triplet to compose the **5** representation. Since the triplet Higgs field induces proton decay, it has to be superheavy, while the doublet Higgs field should have a mass of the electroweak scale. Thus, the doublet and the triplet must be split. In addition, the triplet mass should be much larger than the GUT scale indicated by the GCU, $M_G \sim 10^{16}\text{GeV}$, to sufficiently suppress the proton decay [10] (if we assume no tunings [11]), which would result in rather large threshold corrections.

We have recently proposed an interesting GUT scenario [12], called grand gauge-Higgs unification, in which the unified gauge symmetry is broken via the Hosotani mechanism [13]. This mechanism works in a higher-dimensional gauge theory with non-trivial cycles: the extra-dimensional components of the gauge field acquire non-vanishing vacuum expectation values (VEVs) to break the gauge symmetry. Our main point in the present paper is that the VEV discussed in Ref. [12] which breaks the $SU(5)$ unified group to the SM one is also useful to solve the DT splitting problem. This is because, interestingly, the VEV has a form to be called the “missing VEV” (though in an inverse way) in the sense that it contributes only to the doublet mass and not to the triplet mass. In fact, we will see that when we introduce a bulk field in **5** representation with a certain boundary condition (BC), its doublet component becomes massless on the background while the triplet component has a mass of the compactification scale, and thus the splitting is realized. Note that usually such a form is forbidden by the traceless condition of the adjoint representation of the $SU(5)$ group. In contrast, in the Hosotani mechanism, the adjoint Higgs field appears as the Wilson line phase and its phase nature allows the form. Namely, the Hosotani-breaking plays an essential role in our solution.

Since only non-SUSY examples are discussed in Ref. [12], we examine how we can realize the same VEV in SUSY models, as well as its stability. Fortunately or unfortunately, once the VEV is selected, the situation is very similar to the orbifold GUT models [8] though there are several constraints, *e.g.* the unified symmetry is broken only by the VEV but not by brane localized interactions. For instance, the mass spectra of the bulk fields are controlled by the parity of the hypercharge, and the proton decay through the dimension 4 and 5 operators can be suppressed by an approximate $U(1)_R$ symmetry which is consistent with the superheavy color triplet Higgs field. There is, however, one significant difference, besides a theoretical advantage that the symmetry breaking pattern in our model is well controlled by the calculable

[†] See Refs. [3, 4, 5, 6, 7, 8, 9] for known solutions.

dynamics independent of the ultraviolet completion [12]. An immediately apparent by-product of the supersymmetrization is the existence of light adjoint chiral multiplets with respect to the SM gauge group, as the Wilson line phase gets only a loop induced mass which is suppressed by the SUSY breaking scale and the mass differences relative to its superpartners can not be larger than the SUSY breaking scale. This is a generic and characteristic prediction of our solution[‡]. Although these fields destroy the success of the GCU, it is possible to recover it by introducing additional fields, without making the unified gauge coupling constant non-perturbative below the GUT scale. Some of these additional fields may be also light to be observed in the TeV-scale collider experiments.

This paper is organized as follows. In section 2, we briefly review the non-SUSY grand gauge-Higgs unification. Then we supersymmetrize it to show the DT splitting can be realized without fine-tuning if a certain form of the VEV of the Wilson line phase is assumed. We discuss the stability of the VEV in section 3, which justifies our solution. In section 4, we briefly comment on some related topics. Section 5 is devoted to the summary and discussions.

2 Setup

In this section, we introduce our setup, using the simplest example of a five-dimensional (5D) $SU(5)$ model compactified on an S^1/\mathbb{Z}_2 orbifold with its radius being of the GUT scale. We first review the non-SUSY version discussed in Ref. [12] for illustration purpose, and then supersymmetrize it.

2.1 non-SUSY grand gauge-Higgs unification

At first glance, there is a difficulty in the application of the Hosotani mechanism to GUTs since the massless adjoint fields with respect to the remaining gauge symmetry in the extra-dimensional components tend to be projected out in models that realize the chiral fermions. In Ref. [12], this difficulty is evaded by virtue of the diagonal embedding method [17]. To realize this method in an S^1/\mathbb{Z}_2 model, we prepare two copies of the gauge symmetry which are exchangeable. Namely, we impose $SU(5) \times SU(5) \times \mathbb{Z}_2$ symmetry for our $SU(5)$ model where the \mathbb{Z}_2 action exchanges the two gauge groups. We call the gauge fields for the two $SU(5)$ groups $A_M^{(1)}$ and $A_M^{(2)}$, respectively, where $M = \mu (= 0-3)$, 5 is a 5D Lorentzian index. The BCs around the two endpoints of the S^1/\mathbb{Z}_2 , $y_0 = 0$ and $y_\pi = \pi R$, are given as

$$A_\mu^{(1)}(y_i - y) = A_\mu^{(2)}(y_i + y), \quad A_5^{(1)}(y_i - y) = -A_5^{(2)}(y_i + y), \quad (1)$$

for $i = 0, \pi$, where y is the 5th dimensional coordinate. Defining the eigenstate of the exchange as $X^{(\pm)} = (X^{(1)} \pm X^{(2)})/\sqrt{2}$, we see that $A_\mu^{(+)}$ and $A_5^{(-)}$ obey the Neumann BC at the both endpoints and thus have the zero-modes. In other words, the gauge symmetry of the 4D

[‡] To be more precise, it is a prediction of SUSY grand gauge-Higgs unifications, and is shared with the models with Dirac gaugino masses [14, 15, 16].

effective theory is the diagonal part of $SU(5) \times SU(5)$ (or our GUT symmetry is *embedded* into the *diagonal* part), and an adjoint scalar field is actually obtained.

Since $A_5^{(-)}$ is a part of the gauge fields, it is not a simple adjoint scalar field but composes the Wilson loop

$$W = \mathcal{P} \exp \left(i \int_0^{2L} \frac{g}{\sqrt{2}} A_5^{(-)a} (T_1^a - T_2^a) dy \right) \xrightarrow{\text{on } (\mathbf{5}, \mathbf{1})} \exp (i \text{diag} (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5)), \quad (2)$$

where \mathcal{P} denotes the path-ordered integral, g is the common gauge coupling constant, $L = y_\pi - y_0 = \pi R$ is the length of the extra-dimension, T_i is the generator of each $SU(5)$ symmetry, a is an $SU(5)$ adjoint index, and $\sum \theta_i = 0$. In the last expression, we have used the (remaining) $SU(5)$ rotation to diagonalize $A_5^{(-)}$ and employed the representation acting on the $(\mathbf{5}, \mathbf{1})$ for concreteness, where $\theta_i = \sqrt{2}gL(A_5^{(-)a}T_{\mathbf{5}}^a)_{ii}$ and $T_{\mathbf{5}}$ is the $SU(5)$ generator on $\mathbf{5}$ with the usual normalization $\text{tr}(T_{\mathbf{5}}^a T_{\mathbf{5}}^b) = \delta^{ab}/2^\S$. As evident from the above expression, the VEV (and actually the system itself) is periodic in θ_i .

The form of the VEV which is discussed in Ref. [12] and we are interested in is given by $\theta_1 = \theta_2 = \theta_3 = 2\pi$ and $\theta_4 = \theta_5 = -3\pi$, *i.e.* $\langle W \rangle = \text{diag}(1, 1, 1, -1, -1) \equiv P_W$. This VEV does not affect the triplet component of $\mathbf{5}$ but does affect the doublet to *split* it. This “missing VEV”, which is forbidden for a simple adjoint scalar field by the traceless condition, is allowed thanks to the phase nature of the Hosotani-breaking.

This system with the non-trivial VEV of the Wilson line is known to be equivalent to the one connected by the (broken) gauge transformation, especially with the gauge parameter $\alpha^{(-)} = g \langle A_5^{(-)} \rangle (y - y_0)$ by which the VEV is gauged away [18]. The former is called as the Hosotani basis and the latter as the Scherk-Schwartz (SS) basis where the effect of the VEV appears in the BCs around $y = y_\pi$, modified from Eq. (1) to

$$A_\mu^{(1)}(y_\pi - y) = P_W A_\mu^{(2)}(y_\pi + y) P_W^\dagger, \quad A_5^{(1)}(y_\pi - y) = -P_W A_5^{(2)}(y_\pi + y) P_W^\dagger. \quad (3)$$

With these modified BCs, the $SU(5)$ symmetry is broken down to the SM one, and the SM adjoint components of A_5 have the zero-modes. The components corresponding to the broken generators do not have the zero-modes in this basis, while they are eaten through the Higgs mechanism in the Hosotani basis. Note that the way of the symmetry breaking in the SS picture, *i.e.* by the BCs, is the same as in the orbifold breaking [8]. Actually the situation is quite similar to the orbifold GUTs although with several constraints, *e.g.* on the possible matter content.

For the matter fields, in this paper, we treat only those that are non-singlet of at most one of the gauge group, for simplicity. To be more concrete, we introduce for instance a fermion $\Psi(\mathbf{R}, \mathbf{1})^{(1)}$ with \mathbf{R} being a representation of the $SU(5)$ group[¶]. Then, the exchange \mathbb{Z}_2 symmetry requires its \mathbb{Z}_2 partner $\Psi(\mathbf{1}, \mathbf{R})^{(2)}$ as well. Their BCs are given as

$$\Psi^{(1)}(y_\pi - y) = -\eta_i^\Psi \gamma_5 \Psi^{(2)}(y_\pi + y), \quad (4)$$

[§] The superficial difference of the factor 2 compared with the expression in Ref. [12] comes from the generator.

[¶] For the other case where $\Psi(\mathbf{R}_1, \mathbf{R}_2)$ is introduced, see Ref. [12].

in the Hosotani basis where $\eta_i = \pm 1$ is a parameter associated with each fermion. As one of η_i can be reabsorbed by changing γ_5 , *i.e.* by the charge conjugation, we set $\eta_0 = +1$ and $\eta_\pi = \eta$ hereafter. Then, $\Psi_L^{(+)}$ and $\Psi_R^{(-)}$ have the zero-modes when $\eta = +1$ while none have when $\eta = -1$. Thus, the zero-modes appear always in vector-like pairs of \mathbb{Z}_2 even and odd fields from the bulk fermion, and we put chiral fermions on a brane.

In the SS basis, the BCs become

$$\Psi^{(1)}(y_\pi - y) = -\eta_i^\Psi \gamma_5 W_{\mathbf{R}} \Psi^{(2)}(y_\pi + y), \quad (5)$$

where $W_{\mathbf{R}}$ is the Wilson line phase acting on \mathbf{R} . It is easy to derive $\Psi^{(1)}(y+2L) = \eta W_{\mathbf{R}} \Psi^{(1)}(y)$, and we call the components with $\eta W_{\mathbf{R}} = 1$ (-1) (anti-)periodic.

In particular, for $\mathbf{R} = \mathbf{5}$ with $\eta = -1$, the doublet component has the zero-mode while the triplet does not. Although we can get a massless doublet Higgs scalar field at the tree level in a similar way, the loop corrections likely make it superheavy, and thus we consider SUSY models.

2.2 SUSY version

The same story discussed in the previous subsection can be applied also in SUSY models if we replace all the fields by the corresponding superfields. Thus, once the desired VEV P_W is obtained, the DT splitting is easily realized by introducing a bulk $\mathbf{5}$ hypermultiplet with $\eta = -1$ for the Higgs fields.

Then, the remaining task is to examine when the VEV is realized. Although, according to the literature [19], it is difficult to realize the vacuum as the global minimum, the vacua on a local minima bring no problems if their lifetimes are long enough. Actually, once the universe cools down on a local minimum, in our case, since the length of the “tunnel” is of order $1/R \sim M_G$, the tunneling rate is parametrically suppressed. Thus, we examine only that the vacuum lives on a minimum and do not care if it is the global one or not in the following. For this purpose, we should check if there are no tadpole terms for the fluctuations of θ_i around the desired vacuum, $\delta\theta_i$, and if they are not tachyonic.

Note that $\delta\theta_i$ is odd under the exchange \mathbb{Z}_2 symmetry by the same reason as the so-called H-parity [20]: since the system is invariant under $\theta_i \rightarrow -\theta_i$ and $\theta_i \rightarrow \theta_i + 2\pi$, so is under $\delta\theta_i \rightarrow -\delta\theta_i$ even for $i = 4, 5$ for which $\langle \theta_i \rangle$ is non-trivial as $\theta_i = \pi + \delta\theta_i \sim -(\pi + \delta\theta_i) \sim \pi - \delta\theta_i$ ^{||}. This \mathbb{Z}_2 invariance protects the tadpole terms even though there is a SM singlet chiral multiplet as the adjoint of the $U(1)$ hypercharge which couples both to heavy and light fields, in great contrast to the sliding singlet mechanism applied to $SU(5)$ models [15, 21]**. Then, the question is only the signs of the mass squared of $\delta\theta_i$.

^{||} It is also understood by the transformation of the Wilson line under the \mathbb{Z}_2 action $W \rightarrow W^*$ and the fact that the VEV $\langle W \rangle$ is real and thus invariant under it.

^{**} The other “missing VEV” which affects only the triplet mass, *i.e.* $\theta_1 = \theta_2 = \theta_3 = 4\pi/3$ and $\theta_4 = \theta_5 = -2\pi$, breaks the \mathbb{Z}_2 symmetry and thus huge tadpole term will be induced by the quantum corrections.

3 Stability of the VEV

In this section, we study the mass squared of $\delta\theta_i$ to find if the vacuum is stable or unstable. Since the mass terms are generated only by the loop effects which are vanishing if the SUSY is exact, the signs crucially depend on the SUSY breaking. First, as a simple example, we employ the SS SUSY breaking [22], and then examine the condition for the vacuum to be stable, with general SUSY breaking.

In the SS mechanism, the SUSY is broken by BCs which twist the R -symmetry and thus give different masses for different components of a supermultiplet. By this, the fermion (scalar) component in each gauge (hyper-) multiplet gets SUSY breaking masses β/R with β being a twist parameter, called the SS phase. Although a tiny β seems unnatural since, in the supergravity, this breaking is equivalent to the one by the radion F -term [23], we assume $\beta \sim 10^{-14}$ by hand as we use this mechanism just as an illustrating example.

With this SUSY breaking, the contributions from each periodic and anti-periodic modes to the 1-loop effective potential of the fluctuations $\delta\theta_i$ are given by [24]

$$V^{(+)}(\delta\theta_i) = 2sC \sum_{w=1}^{\infty} \frac{1}{w^5} (1 - \cos(2\pi w\beta)) \cos(wQ_i\delta\theta_i) \sim 2sC \frac{1}{2} (\ln(4\beta^2) - 3) \beta^2 (Q_i\delta\theta_i)^2, \quad (6)$$

$$V^{(-)}(\delta\theta_i) = 2sC \sum_{w=1}^{\infty} \frac{(-1)^w}{w^5} (1 - \cos(2\pi w\beta)) \cos(wQ_i\delta\theta_i) \sim 2sC (\ln 2) \beta^2 (Q_i\delta\theta_i)^2, \quad (7)$$

respectively, where $s = -1$ ($+1$) for the gauge (hyper-) multiplets, $C = 3/(64\pi^7 R^5)$, Q_i is the charge of the relevant mode with respect to the $U(1)$ symmetry corresponding to $\delta\theta_i$, and we have expanded the functions for $\beta \ll 1$ [25] at the last expressions. When $\beta \ll 1$, the contributions to the mass of $\delta\theta_i$ are dominated by those that are enhanced by the factor $\ln(4\beta^2)$ from the periodic modes, to be more precise, from the zero-modes. Given that β is so small, the higher-loop corrections are not so suppressed and we should resum all the leading log terms, by solving the renormalization group equations (RGEs). The other terms are treated as the threshold corrections which are of the next to the leading order. Thus, if we are satisfied with the 1-loop RGE approximation, the mass of $\delta\theta_i$ is controlled only by the 4D effective theory [25]. This observation is apparently not specific to the SS breaking but is applied to the cases with general SUSY breaking.

We shall turn to the RGEs in models with the adjoint chiral multiplets, $(\Sigma_8, \Sigma_3, \Sigma_1)$ for $(SU(3), SU(2), U(1))$ of the SM and possible vector-like pairs for the GCU in addition to the MSSM matter content. The adjoint multiplets interact with the bulk fields via the Yukawa terms in the superpotential given in Ref. [26]. The RGEs can be calculated, for example as in Ref. [27], and we see that the soft mass squared terms of Σ_8 and Σ_3 are pushed up by the gaugino contributions, and thus they are likely positive. On the other hand, Σ_1 is gauge singlet and its soft mass squared is driven only by the Yukawa interactions. Then, to make the contribution positive, some of the soft mass squared of the superfields that take part in the Yukawa interactions have to be negative. It might sound unnatural, but it says just that the fermion component is heavier than the scalar one, and the latter needs not tachyonic. In

fact, for instance, while $\delta\theta_i$ is massless due to the shift symmetry at the tree level, its fermion partner may have a mass of the SUSY breaking scale. This situation is expressed by a μ -like term of the SUSY breaking scale and a soft mass squared term that cancels the contribution of the μ -like term to the $\delta\theta_i$ mass by a negative coefficient, in the superfield formalism [28]. If the supersymmetric mass is larger, as the cases of the supermultiplets with masses of the intermediate scale which may be introduced to recover the GCU, the scalar component is also massive. For these multiplets, the magnitudes of the soft mass squared may be rather large and then the soft masses of $\delta\theta_i$ become large enough to avoid the tachyonic masses against possible B -terms.

In this way, by choosing appropriate SUSY breaking pattern, which does not require fine-tuning, it is possible that all the $\delta\theta_i$ have positive mass squared to make the vacuum stable. As discussed, once the vacuum is realized, its lifetime is parametrically long even if it resides on a local minimum. Then our analysis on the vacuum which leads to the DT splitting even in an $SU(5)$ model is valid.

4 Some related topics

Before closing this paper, we shall comment on the Yukawa couplings, the proton decay, the GCU and the μ -problem. Many of them have a similar feature to the orbifold GUTs [8].

As mentioned above, in our setup, chiral fermions can not be obtained from the bulk fields, and thus we put the three generational quarks and leptons on a brane, where the $SU(5)$ symmetry is not broken. To evade the wrong GUT relation between the down-type quarks and the charged leptons, we have to introduce bulk matter fields as messengers of the $SU(5)$ breaking [8]. For instance, a bulk $\bar{\mathbf{5}}$ hypermultiplet with $\eta = -1$ serves an additional lepton doublet and its vector-like partner as the zero-modes. Since the former (latter) is even (odd) under the Z_2 symmetry that forbids the tadpole term of Σ_1 , they can not be superheavy, while they can be if we introduce a further bulk $\mathbf{5}$ hypermultiplet. In any case, these additional matters can be used to break the wrong relation because some of the zero-modes can couple to the boundary lepton doublets, while there are no additional light right-handed down-type quarks. Similarly, a pair of bulk $\mathbf{10}$ and $\bar{\mathbf{10}}$ could be used.

At this stage, we do not attempt to explain the hierarchical structure of the Yukawa couplings, but just mention that the flavor symmetries [29, 30] do not conflict with our solution. We leave the task to construct a realistic model of the flavor as a future work.

The proton decay via the dimension 4 and 5 operators is efficiently suppressed by an approximate $U(1)_R$ symmetry. The one via the gauge boson exchange is, on one hand, enhanced because all the Kaluza-Klein (KK) modes contribute and their couplings to the boundary fields are larger by the factor $\sqrt{2}$ than those of the corresponding zero-modes due to the normalizations of the 5th dimensional wave functions. On the other hand, if the bulk originated field discussed above dominates a light mode while its $SU(5)$ partner connected by the gauge boson decouples due to the KK mass, it is suppressed. Thus, it highly depends on models of

the flavor and, unfortunately, it seems difficult to derive a generic prediction. These features are shared with the orbifold GUTs [8].

The GCU in the MSSM is neither a prediction in our scenario, since the light adjoint chiral multiplets Σ_1 , Σ_3 and Σ_8 exist, which is characteristic of our scenario. They contribute to the beta function coefficient by $(3, 2, 0)$ for the $(SU(3), SU(2), U(1))$ with the $SU(5)$ normalization of the $U(1)$. To recover the unification, we shall add $(n, 1 + n, 3 + n)$ in some way^{††}. The straightforward possibility is to introduce their $SU(5)$ partners, $(\mathbf{3}, \mathbf{2}, 5/6)$ and $(\bar{\mathbf{3}}, \mathbf{2}, -5/6)$ which leads to $n = 2$. In this case, however, a Landau pole appears below the GUT scale. An example with $n = 1$ which keeps the couplings perturbative, $\alpha_{\text{GUT}} \sim 0.3$ at the 1-loop level, is to introduce two pairs of $(\mathbf{1}, \mathbf{2}, 1/2)$ and each pair of $(\bar{\mathbf{3}}, \mathbf{1}, -2/3)$ and $(\mathbf{1}, \mathbf{1}, 1)$. These pairs can originate from bulk $\mathbf{5}$ multiplets with $\eta = -1$ and $\mathbf{10}$ with $\eta = +1$. Some of these may be identified with the bulk fields needed to break the wrong GUT relation or the messengers of the SUSY breaking.

Finally, we shall discuss the μ -problem. The $U(1)_R$ symmetry that suppresses the proton decay forbids the μ -term, which could explain why the μ -term is of the SUSY breaking scale [8]. In addition, however, when we regard the additional two pairs of the doublet as the matter or messenger fields, the two Higgs doublets should come from one bulk field and have the different \mathbb{Z}_2 parities which also forbid the μ -term in our scenario. Then, since the \mathbb{Z}_2 symmetry should not be broken so strongly to protect the tadpole term, the μ -term may be too suppressed. We may consider that proton decay is suppressed not by the $U(1)_R$ but by the \mathbb{Z}_2 symmetry to avoid it.

We can also consider another case where the \mathbb{Z}_2 symmetry remains unbroken. In this case, one pair of the additional doublet should be regarded as Higgs fields in order to write the μ -term (after the $U(1)_R$ breaking). To write the mass terms for the other additional pairs, we should double them and set their masses to be of the intermediate scale to effectively reproduce the required contributions to the GCU. In this case, the model is a SUSY version of the inert singlet [32]/doublet [33]/triplet [34] coexisting model, with the color octet chiral multiplet in addition, which can decay only through virtual superheavy fields and thus may form the so-called R-hadron.

5 Summary and Discussions

We have presented a new solution to the DT splitting problem in SUSY-GUTs where the unified gauge symmetry is broken through the Hosotani mechanism [13]. An adjoint chiral multiplet is obtained by the diagonal embedding method [17, 12] on an orbifold. Thanks to the fact that it is not an simple adjoint field but composes the Wilson line phase, the “missing VEV” is allowed even in $SU(5)$ models, which is a key ingredient of our solution. We have discussed that even if the vacuum resides on a local minimum, the lifetime is parametrically long and then we examined the stability of the vacuum. We have showed that once the VEV is realized, fortunately or unfortunately, our scenario basically reduces to the orbifold GUTs [8]

^{††} It is possible that the unification scale is also modified [16].

with several constraints. Thus, some discussions, *e.g.* on the Yukawa couplings and the proton decay, are shared in these scenarios.

A distinctive difference appears in the possible matter content in the 4D effective theories. In particular, our scenario predicts light adjoint chiral multiplets with respect to the SM gauge group which are testable at TeV-scale collider experiments. Although these light fields disturb the GCU, it can be recovered by introducing additional vector-like fields, keeping the unified gauge coupling constant perturbative. Depending on the masses of the additional colored particles, the lifetime of the color octet multiplet may become rather long to form the so-called R-hadron.

In addition, our models contain a \mathbb{Z}_2 symmetry under which the light adjoint multiplets change the sign. The \mathbb{Z}_2 symmetry may be broken slightly and may remain exact. The phenomenology of the colorless fields would largely depend on the presence of the breaking. For example, when the \mathbb{Z}_2 symmetry remains unbroken, in order to write the μ -term, two inert doublet fields [33] should appear, in addition to the inert singlet [32] and triplet [34]. We will study their phenomenology in another place [31].

Finally, we comment on the application to the string theory. The idea of the diagonal embedding [17] was proposed in the context of the string theory in the first place. Thus, our field theoretical setup may be regarded as an effective theory of such string models. Although, in this view, we neglect all the stringy effects which could modify our discussion especially when the compactification scale is not smaller than the string scale, it may be possible that our solution can be applied also to such string models.

Acknowledgments

We appreciate M. Kakizaki, S. Kenemura, K. Kojima, Y. Nakai and K. Takenaga for valuable discussions.

References

- [1] H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32** (1974) 438.
- [2] E. Witten, Nucl. Phys. B **188** (1981) 513; S. Dimopoulos, S. Raby and F. Wilczek, Phys. Rev. D **24** (1981) 1681; S. Dimopoulos and H. Georgi, Nucl. Phys. B **193** (1981) 150; N. Sakai, Z. Phys. C **11** (1981) 153.
- [3] E. Witten, Phys. Lett. B **105** (1981) 267; D. V. Nanopoulos and K. Tamvakis, Phys. Lett. B **113** (1982) 151; S. Dimopoulos and H. Georgi, Phys. Lett. B **117** (1982) 287; K. Tabata, I. Umemura and K. Yamamoto, Prog. Theor. Phys. **71** (1984) 615; A. Sen, Phys. Lett. B **148** (1984) 65; G. R. Dvali, Phys. Lett. B **324** (1994) 59; S. M. Barr, Phys. Rev. D **57** (1998) 190; N. Maekawa and T. Yamashita, Phys. Rev. D **68** (2003) 055001.

- [4] H. Georgi, Phys. Lett. B **108** (1982) 283; A. Masiero, D. V. Nanopoulos, K. Tamvakis and T. Yanagida, Phys. Lett. B **115** (1982) 380; B. Grinstein, Nucl. Phys. B **206** (1982) 387; K. S. Babu, I. Gogoladze and Z. Tavartkiladze, Phys. Lett. B **650** (2007) 49.
- [5] S. M. Barr, Phys. Lett. B **112** (1982) 219; I. Antoniadis, J. R. Ellis, J. S. Hagelin and D. V. Nanopoulos, Phys. Lett. B **194** (1987) 231; N. Maekawa and T. Yamashita, Phys. Lett. B **567** (2003) 330.
- [6] S. Dimopoulos and F. Wilczek, Print-81-0600 (SANTA BARBARA), NSF-ITP-82-07; M. Srednicki, Nucl. Phys. B **202** (1982) 327; K. S. Babu and S. M. Barr, Phys. Rev. D **48** (1993) 5354; S. M. Barr and S. Raby, Phys. Rev. Lett. **79** (1997) 4748; N. Maekawa, Prog. Theor. Phys. **106** (2001) 401; N. Maekawa and T. Yamashita, Prog. Theor. Phys. **107** (2002) 1201; *ibid* **110** (2003) 93.
- [7] K. Inoue, A. Kakuto and H. Takano, Prog. Theor. Phys. **75** (1986) 664; A. A. Anselm and A. A. Johansen, Phys. Lett. B **200** (1988) 331; A. A. Anselm, Sov. Phys. JETP **67** (1988) 663 [Zh. Eksp. Teor. Fiz. **94** (1988) 26]; Z. G. Berezhiani and G. R. Dvali, Bull. Lebedev Phys. Inst. **5** (1989) 55 [Kratk. Soobshch. Fiz. **5** (1989) 42].
- [8] Y. Kawamura, Prog. Theor. Phys. **103**, 613 (2000); *ibid* **105**, 691 (2001); *ibid* **105**, 999 (2001); L. J. Hall and Y. Nomura, Phys. Rev. D **64** (2001) 055003; *ibid* **65** (2002) 125012; *ibid* **66** (2002) 075004.
- [9] M. Kakizaki and M. Yamaguchi, Prog. Theor. Phys. **107** (2002) 433; N. Maru, Phys. Lett. B **522** (2001) 117.
- [10] T. Goto and T. Nihei, Phys. Rev. D **59** (1999) 115009; H. Murayama and A. Pierce, Phys. Rev. D **65** (2002) 055009.
- [11] B. Bajc, P. Fileviez Perez and G. Senjanovic, Phys. Rev. D **66** (2002) 075005; D. Emmanuel-Costa and S. Wiesenfeldt, Nucl. Phys. B **661** (2003) 62.
- [12] K. Kojima, K. Takenaga and T. Yamashita, arXiv:1103.1234 [hep-ph].
- [13] Y. Hosotani, Phys. Lett. B **126**, 309 (1983); *ibid* **129**, 193 (1983); Phys. Rev. D **29**, 731 (1984). Ann. of Phys. **190**, 233 (1989);
- [14] P. Fayet, Phys. Lett. B **78** (1978) 417.
- [15] J. Polchinski and L. Susskind, Phys. Rev. D **26** (1982) 3661. 5
- [16] A. E. Nelson, N. Rius, V. Sanz and M. Unsal, JHEP **0208** (2002) 039; K. Benakli and M. D. Goodsell, Nucl. Phys. B **840** (2010) 1
- [17] K. R. Dienes and J. March-Russell, Nucl. Phys. B **479** (1996) 113; D. C. Lewellen, Nucl. Phys. B **337**, 61 (1990); G. Aldazabal, A. Font, L. E. Ibanez and A. M. Uranga, Nucl. Phys. B **452**, 3 (1995); J. Erler, Nucl. Phys. B **475**, 597 (1996); Z. Kakushadze and S. H. H. Tye, Phys. Rev. D **55**, 7878 (1997); Phys. Rev. D **55**, 7896 (1997); M. Ito *et al.*, Phys. Rev. D **83** (2011) 091703; arXiv:1104.0765 [hep-th].

- [18] N. Haba, M. Harada, Y. Hosotani and Y. Kawamura, Nucl. Phys. B **657** (2003) 169 [Erratum-ibid. B **669** (2003) 381].
- [19] A. T. Davies and A. McLachlan, Phys. Lett. B **200**, 305 (1988); Nucl. Phys. B **317**, 237 (1989).
- [20] Y. Hosotani, P. Ko and M. Tanaka, Phys. Lett. B **680** (2009) 179; Y. Hosotani, M. Tanaka and N. Uekusa, Phys. Rev. D **82** (2010) 115024.
- [21] A. Ali and I. Montvay, Phys. Lett. B **124** (1983) 237; A. B. Lahanas, Phys. Lett. B **124** (1983) 341.
- [22] J. Scherk and J. H. Schwarz, Phys. Lett. B **82** (1979) 60; Nucl. Phys. B **153** (1979) 61.
- [23] Z. Chacko and M. A. Luty, JHEP **0105** (2001) 067; D. Marti and A. Pomarol, Phys. Rev. D **64** (2001) 105025; G. von Gersdorff and M. Quiros, Phys. Rev. D **65** (2002) 064016; G. von Gersdorff, M. Quiros and A. Riotto, Nucl. Phys. B **634** (2002) 90.
- [24] K. Takenaga, Phys. Rev. D **58** (1998) 026004 [Erratum-ibid. D **61** (2000) 129902]; N. Haba, Y. Hosotani, Y. Kawamura and T. Yamashita, Phys. Rev. D **70**, 015010 (2004); N. Haba and T. Yamashita, JHEP **0402** (2004) 059.
- [25] N. Haba, K. Takenaga and T. Yamashita, Phys. Lett. B **615**, 247 (2005); N. Haba, S. Matsumoto, N. Okada and T. Yamashita, JHEP **0602**, 073 (2006).
- [26] N. Arkani-Hamed, T. Gregoire and J. G. Wacker, JHEP **0203** (2002) 055.
- [27] F. Borzumati and T. Yamashita, Prog. Theor. Phys. **124** (2010) 761.
- [28] G. Burdman and Y. Nomura, Nucl. Phys. B **656** (2003) 3.
- [29] L. E. Ibanez and G. G. Ross, Phys. Lett. B **332** (1994) 100; P. Binetruiy and P. Ramond, Phys. Lett. B **350** (1995) 49; E. Dudas, S. Pokorski and C. A. Savoy, Phys. Lett. B **356** (1995) 45; C. H. Albright and S. Nandi, Mod. Phys. Lett. A **11** (1996) 737; Phys. Rev. D **53** (1996) 2699.
- [30] F. Wilczek and A. Zee, Phys. Lett. B **70** (1977) 418 [Erratum-ibid. **72B** (1978) 504]; S. Pakvasa and H. Sugawara, Phys. Lett. B **73** (1978) 61; A. De Rujula, H. Georgi and S. L. Glashow, Annals Phys. **109** (1977) 258.
- [31] M. Kakizaki, S. Kanemura, H. Taniguchi and T. Yamashita, in preparation.
- [32] V. Silveira and A. Zee, Phys. Lett. B **161** (1985) 136; C. P. Burgess, M. Pospelov and T. ter Veldhuis, Nucl. Phys. B **619** (2001) 709.
- [33] E. Ma, Phys. Rev. D **73** (2006) 077301; R. Barbieri, L. J. Hall and V. S. Rychkov, Phys. Rev. D **74** (2006) 015007.
- [34] T. Araki, C. Q. Geng and K. I. Nagao, Phys. Rev. D **83** (2011) 075014.